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A Critical Look at γ Determinations from $B \to \pi K$ Decays

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Abstract

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A CRITICAL LOOK AT γ DETERMINATIONS FROM $B \to \pi K$ DECAYS

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The determination of the angle γ of the unitarity triangle of the CKM matrix is a challenge for the B-factories. In this context, $B \to \pi K$ decays received a lot of attention, providing various interesting ways to constrain and determine γ. These strategies are briefly reviewed, and their virtues and weaknesses are compared with one another.

1 Setting the Scene

In order to obtain direct information on the angle γ of the unitarity triangle of the CKM matrix in an experimentally feasible way, $B \to \pi K$ decays appear very promising. Fortunately, experimental data on these modes are now starting to become available. In 1997, the CLEO collaboration reported the first results on the decays $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$; last year, the first observation of $B^\pm \to \pi^0 K^\pm$ was announced.[1] So far, only results for CP-averaged branching ratios have been reported, with values at the 10^{-5} level and large experimental uncertainties. However, already such CP-averaged branching ratios may lead to highly non-trivial constraints on γ.[2] The following three combinations of $B \to \pi K$ decays were considered in the literature: $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$,[2-4] $B^\pm \to \pi^\pm K$ and $B^\pm \to \pi^0 K^\pm$,[5-7] as well as the combination of the neutral decays $B_d \to \pi^0 K$ and $B_d \to \pi^\mp K^\pm$.[7]

2 Probing γ with $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$

Within the framework of the Standard Model, the most important contributions to these decays originate from QCD penguin topologies. Making use of the $SU(2)$ isospin symmetry of strong interactions, we obtain

$$A(B^+ \to \pi^+ K^0) \equiv P, \quad A(B_d^0 \to \pi^- K^+) = -\left[P + T + P_{\rm ew}^{\rm C}\right], \tag{1}$$

where

$$T \equiv |T| e^{i\delta_T} e^{i\gamma} \quad \text{and} \quad P_{\rm ew}^{\rm C} \equiv -\left|P_{\rm ew}^{\rm C}\right| e^{i\delta_{\rm ew}^{\rm C}} \tag{2}$$

are due to tree-diagram-like topologies and electroweak (EW) penguins, respectively. The label "C" reminds us that only "colour-suppressed" EW penguin topologies contribute to $P_{\rm ew}^{\rm C}$. Making use of the unitarity of the CKM

matrix and applying the Wolfenstein parametrization yields

$$P \equiv A(B^+ \to \pi^+ K^0) = -\left(1-\frac{\lambda^2}{2}\right)\lambda^2 A\left[1+\rho\, e^{i\theta} e^{i\gamma}\right]\mathcal{P}_{tc}\,, \tag{3}$$

where

$$\rho\, e^{i\theta} = \frac{\lambda^2 R_b}{1-\lambda^2/2}\left[1-\left(\frac{\mathcal{P}_{uc}+\mathcal{A}}{\mathcal{P}_{tc}}\right)\right], \tag{4}$$

and $\lambda \equiv |V_{us}|$, $A \equiv |V_{cb}|/\lambda^2$, $R_b \equiv |V_{ub}/(\lambda V_{cb})|$. Note that ρ is strongly CKM-suppressed by $\lambda^2 R_b \approx 0.02$. In the parametrization of the $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ observables, it turns out to be very useful to introduce

$$r \equiv \frac{|T|}{\sqrt{\langle |P|^2\rangle}}\,, \quad \epsilon_{\rm C} \equiv \frac{|P_{\rm ew}^{\rm C}|}{\sqrt{\langle |P|^2\rangle}}\,, \tag{5}$$

with $\langle |P|^2\rangle \equiv (|P|^2 + |\overline{P}|^2)/2$, as well as the strong phase differences

$$\delta \equiv \delta_T - \delta_{tc}\,, \quad \Delta_{\rm C} \equiv \delta_{\rm ew}^{\rm C} - \delta_{tc}\,. \tag{6}$$

In addition to the ratio

$$R \equiv \frac{\mathrm{BR}(B_d \to \pi^\mp K^\pm)}{\mathrm{BR}(B^\pm \to \pi^\pm K)} \tag{7}$$

of CP-averaged $B \to \pi K$ branching ratios, also the "pseudo-asymmetry"

$$A_0 \equiv \frac{\mathrm{BR}(B_d^0 \to \pi^- K^+) - \mathrm{BR}(\overline{B_d^0} \to \pi^+ K^-)}{\mathrm{BR}(B^+ \to \pi^+ K^0) + \mathrm{BR}(B^- \to \pi^- \overline{K^0})} \tag{8}$$

plays an important role to probe γ. Explicit expressions for R and A_0 in terms of the parameters specified above are given in Ref. 8. So far, the only available experimental result from the CLEO collaboration is for R:[1]

$$R = 0.9 \pm 0.4 \pm 0.2 \pm 0.2, \tag{9}$$

and no CP-violating effects have been reported. However, if in addition to R also the pseudo-asymmetry A_0 can be measured, it is possible to eliminate the strong phase δ in the expression for R, and to fix contours in the $\gamma - r$ plane,[8] which correspond to the mathematical implementation of a simple triangle construction.[3] In order to determine γ, the quantity r, i.e. the magnitude of the "tree" amplitude T, has to be fixed. At this step, a certain model dependence enters. Since the properly defined amplitude T does not receive contributions only from colour-allowed "tree" topologies, but also from penguin and annihilation processes,[8,9] it may be shifted sizeably from its "factorized" value. Consequently, estimates of the uncertainty of r using

the factorization hypothesis, yielding typically $\Delta r = \mathcal{O}(10\%)$, may be too optimistic.

Interestingly, it is possible to derive bounds on γ that do *not* depend on r at all.[2] To this end, we eliminate again δ in R through A_0. If we now treat r as a "free" variable, we find that R takes the following minimal value:[8]

$$R_{\min} = \kappa\, \sin^2\gamma \,+\, \frac{1}{\kappa}\left(\frac{A_0}{2\,\sin\gamma}\right)^2 \geq \kappa\, \sin^2\gamma\,. \tag{10}$$

Here, the quantity

$$\kappa = \frac{1}{w^2}\left[\,1 + 2\,(\epsilon_{\rm C}\, w)\cos\Delta + (\epsilon_{\rm C}\, w)^2\,\right], \tag{11}$$

with $w = \sqrt{1 + 2\,\rho\,\cos\theta\cos\gamma + \rho^2}$, describes rescattering and EW penguin effects. An allowed range for γ is related to $R_{\min}$, since values of γ implying $R_{\rm exp} < R_{\min}$ are excluded. In particular, $A_0 \neq 0$ would allow us to exclude a certain range of γ around 0° or 180°, whereas a measured value of $R < 1$ would exclude a certain range around 90°, which would be of great phenomenological importance. The first results reported by CLEO in 1997 gave $R = 0.65 \pm 0.40$, whereas the most recent update is that given in (9).

The theoretical accuracy of these constraints on γ is limited both by rescattering processes of the kind $B^+ \to \{\pi^0 K^+, \pi^0 K^{*+}, \ldots\}$,[10,11] and by EW penguin effects.[4,11] The rescattering effects, which may lead to values of $\rho = \mathcal{O}(0.1)$, can be controlled in the contours in the γ–r plane and the associated constraints on γ through experimental data on $B^\pm \to K^\pm K$ decays, the U-spin counterparts of $B^\pm \to \pi^\pm K$.[8,12] Another important indicator for large rescattering effects is provided by $B_d \to K^+ K^-$ modes, for which there already exist stronger experimental constraints.[13]

An improved description of the EW penguins is possible if we use the general expressions for the corresponding four-quark operators, and perform appropriate Fierz transformations. Following these lines,[8,11] we arrive at

$$\frac{\epsilon_{\rm C}}{r}\, e^{i(\Delta_{\rm C} - \delta)} = 0.66 \times \left[\frac{0.41}{R_b}\right] \times a_{\rm C}\, e^{i\omega_{\rm C}}, \tag{12}$$

where $a_{\rm C}\, e^{i\omega_{\rm C}} = a_2^{\rm eff}/a_1^{\rm eff}$ is the ratio of certain generalized "colour factors". Experimental data on $B \to D^{(*)}\pi$ decays imply $a_2/a_1 = \mathcal{O}(0.25)$. However, "colour suppression" in $B \to \pi K$ modes may in principle be different from that in $B \to D^{(*)}\pi$ decays, in particular in the presence of large rescattering effects.[11] A first step to fix the hadronic parameter $a_{\rm C}\, e^{i\omega_{\rm C}}$ experimentally is provided by the mode $B^+ \to \pi^+\pi^0$. Detailed discussions of the impact of rescattering and EW penguin effects on the strategies to probe γ with $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ decays can be found in Refs. 7, 8 and 12.

3 Probing γ with $B^\pm \to \pi^\pm K$ and $B^\pm \to \pi^0 K^\pm$

Several years ago, Gronau, Rosner and London proposed an interesting $SU(3)$ strategy to determine γ with the help of $B^\pm \to \pi^\pm K$, $\pi^0 K^\pm$, $\pi^0\pi^\pm$ decays.[5] However, as was pointed out by Deshpande and He,[14] this elegant approach is unfortunately spoiled by EW penguins, which play an important role in several non-leptonic B-meson decays because of the large top-quark mass.[15] Recently, this approach was resurrected by Neubert and Rosner,[6] who pointed out that the EW penguin contributions can be controlled in this case by using only the general expressions for the corresponding four-quark operators, appropriate Fierz transformations, and the $SU(3)$ flavour symmetry (see also Ref. 3). Since a detailed presentation of these strategies can be found in Ref. 16, we will just have a brief look at their most interesting features.

In the case of $B^+ \to \pi^+ K^0$, $\pi^0 K^+$, the $SU(2)$ isospin symmetry implies

$$A(B^+ \to \pi^+ K^0) \,+\, \sqrt{2}\, A(B^+ \to \pi^0 K^+) = -\left[(T+C) \,+\, P_{\rm ew}\right]. \tag{13}$$

The phase stucture of this relation, which has no $I = 1/2$ piece, is completely analogous to the $B^+ \to \pi^+ K^0$, $B_d^0 \to \pi^- K^+$ case (see (1)):

$$T + C = |T+C|\, e^{i\delta_{T+C}}\, e^{i\gamma}, \quad P_{\rm ew} = -\,|P_{\rm ew}| e^{i\delta_{\rm ew}}\,. \tag{14}$$

In order to probe γ, it is useful to introduce observables $R_{\rm c}$ and $A_0^{\rm c}$ corresponding to R and A_0;[7] their general expressions can be otained from those for R and A_0 by making the following replacements:

$$r \to r_{\rm c} \equiv \frac{|T+C|}{\sqrt{\langle |P|^2\rangle}}\,, \quad \delta \to \delta_{\rm c} \equiv \delta_{T+C} - \delta_{tc}\,, \quad P_{\rm ew}^{\rm C} \to P_{\rm ew}. \tag{15}$$

The measurement of $R_{\rm c}$ and $A_0^{\rm c}$ allows us to fix contours in the γ–$r_{\rm c}$ plane in complete analogy to the $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ strategy. There are, however, important differences from the theoretical point of view. First, the $SU(3)$ symmetry allows us to fix $r_c \propto |T+C|$:[5]

$$T + C \approx -\sqrt{2}\,\frac{V_{us}}{V_{ud}}\,\frac{f_K}{f_\pi}\, A(B^+ \to \pi^+\pi^0)\,, \tag{16}$$

where r_c thus determined is – in contrast to r – not affected by rescattering effects. Second, in the strict $SU(3)$ limit, we have[6]

$$\left|\frac{P_{\rm ew}}{T+C}\right|\, e^{i(\delta_{\rm ew} - \delta_{T+C})} = 0.66 \times \left[\frac{0.41}{R_b}\right]. \tag{17}$$

In contrast to (12), this expression does not involve a hadronic parameter.

The contours in the γ–r_c plane may be affected – in analogy to the $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ case – by rescattering effects.[7] They can be taken

into account with the help of additional data.[8,12,17] The major theoretical advantage of the $B^+ \to \pi^+ K^0$, $\pi^0 K^+$ strategy with respect to $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ is that r_c and $P_{\rm ew}/(T+C)$ can be fixed by using only $SU(3)$ arguments. Consequently, the theoretical accuracy is mainly limited by non-factorizable $SU(3)$-breaking effects.

4 Probing γ with $B_d \to \pi^0 K$ and $B_d \to \pi^\mp K^\pm$

The strategies to probe γ that are allowed by the observables of $B_d \to \pi^0 K$, $\pi^\mp K^\pm$ are completely analogous to the $B^\pm \to \pi^\pm K$, $\pi^0 K^\pm$ case.[7] However, if we require that the neutral kaon be observed as a $K_{\rm S}$, we have an additional observable at our disposal, which is provided by "mixing-induced" CP violation in $B_d \to \pi^0 K_{\rm S}$ and allows us to take into account the rescattering effects in the extraction of γ.[7] To this end, time-dependent measurements are required. The theoretical accuracy of the neutral strategy is only limited by non-factorizable $SU(3)$-breaking corrections, which affect $|T+C|$ and $P_{\rm ew}$.

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